YOUR PRACTICE PAPER

APPLICATIONS AND INTERPRETATION

STANDARD LEVEL
FOR IBDP MATHEMATICS

ANSWERS

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- 4 Sets of Practice Papers
- Distributions of Exam Topics
- Exam Format Analysis
- Comprehensive Formula List

SE PRODUCTION LIMITED

AI SL Practice Set 1 Paper 1 Solution

1. (a) The area of the rectangle

$= 462000000 \text{ cm}^2$

$= 4.62 \times 10^8 \text{ cm}^2$ A2 N2

[2]

(b) The percentage error

$= \left| \frac{450000000 - 462000000}{462000000} \right| \times 100\%$ (A1) for substitution

$= 2.597402597\%$

$= 2.60\%$ A1 N2

[2]

2. (a) $u_{10} = 181$

$\therefore 100 + (10-1)d = 181$ (A1) for correct equation

$9d = 81$

$d = 9$ A1 N2

[2]

(b) 208 A1 N1

[1]

(c) The total number of seats

$= \frac{15}{2}\left[2(100) + (15-1)(9)\right]$ (A1) for substitution

$= 2445$ A1 N2

[2]

3. (a) The mean ball speed

$= \frac{80+76+100+66+40+116+90+76}{8}$ (A1) for correct formula

$= 80.5 \text{ kmh}^{-1}$ A1 N2

[2]

(b) (i) 78 kmh^{-1} A1 N1

(ii) 21.3 kmh^{-1} A1 N1

(iii) 76 kmh^{-1} A1 N1

[3]

4. (a) $y > 250$ — (M1) for setting inequality

$20x > 250$

$x > 12.5$

Thus, the minimum number of tickets is 13. — A1 N2

[2]

(b) $y = 90 + 5x$ — A1 N1

[1]

(c) $20x = 90 + 5x$ — (M1) for setting equation

$15x = 90$

$x = 6$ — (A1) for correct value

The amount of money

$= 20(6)$

$= 120$ USD — A1 N3

[3]

5. (a) (i) $x = 5$ — A2 N2

(ii) $y = 4$ — A2 N2

[4]

(b) $f(x) = 0$

$\frac{2-4x}{5-x} = 0$ — (M1) for setting equation

$2 - 4x = 0$

$2 = 4x$

$x = \frac{1}{2}$ — A1 N2

[2]

6. (a) H_0: The gender and the teaching subjects are independent. A1 N1

[1]

(b) The expected number

$= \dfrac{(35+10+65+45)(10+35)}{300}$ A1

$= \dfrac{(155)(45)}{300}$

$= 23.25$ AG N0

[1]

(c) The p-value

$= 0.00002306699185$ (A1) for correct value

$= 0.0000231$ A1 N2

[2]

(d) The null hypothesis is rejected. A1

As the p-value is less than 5%. R1 N2

[2]

7. (a) (i) $r = \dfrac{3}{4}$ A1 N1

(ii) $u_4 = 10368$ A1 N1

[2]

(b) $u_7 = 24576\left(\dfrac{3}{4}\right)^{7-1}$ (M1) for substitution

$u_7 = 4374$

$u_8 = 24576\left(\dfrac{3}{4}\right)^{8-1}$

$u_8 = 3280.5$

Thus, the smallest term in the sequence that is an integer is $u_7 = 4374$. A1 N2

[2]

(c) S_{27}

$= \dfrac{24576\left(\left(\dfrac{3}{4}\right)^{27} - 1\right)}{\dfrac{3}{4} - 1}$ (A1) for substitution

$= 98262.38736$

$= 98300$ A1 N2

[2]

8. (a) The expected number

$= (13)(0.25)$ (A1) for substitution

$= 3.25$ A1 N2

[2]

(b) The variance

$= (13)(0.25)(1-0.25)$ (A1) for substitution

$= 2.4375$ A1 N2

[2]

(c) The required probability

$= \binom{13}{8}(0.25)^8(1-0.25)^{13-8}$ (A1) for substitution

$= 0.0046602041$

$= 0.00466$ A1 N2

[2]

9. (a) $\cos A\hat{B}C = \frac{AB^2 + BC^2 - AC^2}{2(AB)(BC)}$ (M1) for cosine rule

$\cos A\hat{B}C = \frac{28^2 + 41^2 - 32^2}{2(28)(41)}$ (A1) for substitution

$\cos A\hat{B}C = 0.6276132404$

$A\hat{B}C = 51.12574956^\circ$

$A\hat{B}C = 51.1^\circ$ A1 N3

[3]

(b) The area of the park

$= \frac{1}{2}(AB)(BC)\sin A\hat{B}C$ (M1) for area formula

$= \frac{1}{2}(28)(41)\sin 51.12574956^\circ$ (A1) for substitution

$= 446.873514 \text{ m}^2$

$= 447 \text{ m}^2$ A1 N3

[3]

10. (a) (i) The gradient of L

$= -1 \div \frac{5-1}{7-5}$ (M1) for valid approach

$= -1 \div 2$

$= -\frac{1}{2}$ A1 N2

(ii) The equation of L:

$y - 4 = -\frac{1}{2}(x-4)$ (M1) for substitution

$y = -\frac{1}{2}x + 6$ A1 N2

[4]

(b) Kimberly's office is on the boundary separating the Voronoi cells of the restaurant B and the restaurant C, which is equidistant to them. R1 N1

[1]

11. (a) By TVM Solver:

N = 120
I% = 3.3
PV = 950000
PMT = ?
FV = 0
P / Y = 12
C / Y = 12
PMT : END

(M1)(A1) for correct values

PMT = −9305.412721

Thus, the amount of monthly payment is $9310. A1 N3

[3]

(b) The total amount to be paid

$= (9305.412721)(120)$ (M1) for valid approach

$= \$1116649.527$

$= \$1120000$ A1 N2

[2]

(c) The amount of interest paid

$= 1116649.527 - 950000$ (M1) for valid approach

$= \$166649.5265$

$= \$167000$ A1 N2

[2]

12. (a) The amount of bacteria

$= 100 \times 2^8$ (A1) for correct approach

$= 25600$ A1 N2

[2]

(b) (i) $100 = a \times b^0$ (M1) for setting equation

$a = 100$ A1 N2

(ii) $25600 = 100 \times b^{24}$ (M1) for setting equation

$b^{24} = 256$

$b^{24} - 256 = 0$

By considering the graph of

$y = b^{24} - 256$, $b = 1.259921$.

$\therefore b = 1.26$ A1 N2

[4]

13. (a) $a = 1$, $b = \pi^{-0.1}$ A2 N2

[2]

(b) The estimate of $\int_0^{0.5} f(x)\mathrm{d}x$

$= \frac{1}{2}(0.1)\left[1 + \pi^{-0.5} + 2(\pi^{-0.1} + \pi^{-0.2} + \pi^{-0.3} + \pi^{-0.4})\right]$ (A2) for substitution

$= 0.3811259104$

$= 0.381$ A1 N3

[3]

(c) Overestimate A1 N1

[1]

14. (a) 150 A1 N1

[1]

(b) 15 A1 N1

[1]

(c) $y = a(x - (-5))(x - 15)$ (A1) for correct approach

$y = a(x + 5)(x - 15)$

$150 = a(0 + 5)(0 - 15)$

$150 = -75a$

$a = -2$ A1 N2

$\therefore y = -2(x + 5)(x - 15)$

$y = -2(x^2 - 10x - 75)$ (A1) for correct approach

$y = -2x^2 + 20x + 150$

$\therefore b = 20$ A1 N2

[4]

AI SL Practice Set 1 Paper 2 Solution

1. (a) $3x+y-10$

$=3(3)+1-10$ A1

$=0$

Thus, P lies on L_1. AG N0

[1]

(b) 10 A1 N1

[1]

(c) (i) The coordinates of M

$=\left(\frac{3+11}{2}, \frac{1+(-3)}{2}\right)$ (A1) for substitution

$=(7,-1)$ A1 N2

(ii) The gradient of PQ

$=\frac{-3-1}{11-3}$ (A1) for substitution

$=-\frac{1}{2}$ A1 N2

(iii) The distance between P and Q

$=\sqrt{(11-3)^2+(-3-1)^2}$ (A1) for substitution

$=8.94427191$

$=8.94$ A1 N2

[6]

(d) The gradient of L_1

$=-\frac{3}{1}$

$=-3$ A1

$\because -3\times-\frac{1}{2}$ M1

$=\frac{3}{2}$

$\neq -1$

Thus, L_1 and L_2 are not perpendicular. AG N0

[2]

(e) The gradient of L_3

$= \frac{-1}{-3}$ M1

$= \frac{1}{3}$ A1

The equation of L_3:

$y - 1 = \frac{1}{3}(x - 3)$ A1

$3y - 3 = x - 3$ A1

$x - 3y = 0$ AG N0

[4]

(f) The coordinates of S are $(0, 0)$. (A1) for correct value

The area of the triangle PRS

$= \frac{(10-0)(3-0)}{2}$ (M1) for valid approach

$= 15$ A1 N3

[3]

2. (a) The required probability

$= \mathrm{P}(W < 400)$ — (M1) for valid approach

$= 0.7791219069$

$= 0.779$ — A1 N2

[2]

(b) The expected number

$= (800)(0.7791219069)$ — (A1) for substitution

$= 623.2975255$

$= 623$ — A1 N2

[2]

(c) The required probability

$= \mathrm{P}(W < 385 \mid W < 400)$ — (M1) for valid approach

$= \dfrac{\mathrm{P}(W < 385 \cap W < 400)}{\mathrm{P}(W < 400)}$

$= \dfrac{\mathrm{P}(W < 385)}{\mathrm{P}(W < 400)}$ — (A1) for correct approach

$= 0.4495589773$

$= 0.450$ — A1 N3

[3]

(d) (i) 390 — A1 N1

(ii) 30% — A1 N1

(iii) $\mathrm{P}(W > k) = 0.2$ — (M1) for valid approach

$\mathrm{P}(W < k) = 0.8$

$k = 400.941076$

$k = 401$ — A1 N2

[4]

(e) The expected daily income

$= 800((4)(50\%) + (4.5)(30\%) + (5)(20\%))$ — (A2) for correct approach

$= \$3480$ — A1 N3

[3]

3.	(a)	(i)	$a = 14.02298851$		
			$a = 14.0$	A1	N1
			$b = -420.2413793$		
			$b = -420$	A1	N1
		(ii)	The estimated pulse rate		
			$= 14.02298851(37) - 420.2413793$	(A1) for substitution	
			$= 98.60919557$ beats per minute		
			$= 98.6$ beats per minute	A1	N2
					[4]
	(b)	(i)	$r = 0.592701087$		
			$r = 0.593$	A1	N1
		(ii)	Moderate, Positive	A2	N2
					[3]
	(c)	(i)	H_0: The number of students in each range of pulse rates are evenly distributed.	A1	N1
		(ii)	p-value $= 0.0166229271$	(A1) for correct value	
			p-value $= 0.0166$	A1	N2
		(iii)	The null hypothesis is rejected.	A1	
			As p-value < 0.05.	R1	N2
					[5]
	(d)	(i)	$H_1: \mu_A \neq \mu_B$	A1	N1
		(ii)	p-value $= 0.3065878383$	(A1) for correct value	
			p-value $= 0.307$	A1	N2
		(iii)	The null hypothesis is not rejected.	A1	
			As p-value > 0.01.	R1	N2
					[5]

4.	(a)	(i)	$y = 20 - 4x$	A1 N1	
		(ii)	$0 < x < 5$	A1 N1	
					[2]
	(b)	$V = (4x)(2x)(20 - 4x)$		(M1) for valid approach	
		$V = 8x^2(20 - 4x)$			
		$V = 160x^2 - 32x^3$		A1 N2	
					[2]
	(c)	(i)	By considering the graph of $V = 160x^2 - 32x^3$, the coordinates of the maximum point are $(3.3333342, 592.59259)$.	(M1) for valid approach	
			Thus, the maximum volume is 593 cm^3.	A1 N2	
		(ii)	3.33	A1 N1	
		(iii)	$y = 20 - 4(3.3333342)$	(M1) for substitution	
			$y = 6.6666632$		
			$y = 6.67$	A1 N2	
					[5]
	(d)	$A = 2(4x)(2x) + 2(4x)(20 - 4x) + 2(2x)(20 - 4x)$		(M1) for valid approach	
		$A = 16x^2 + 160x - 32x^2 + 80x - 16x^2$			
		$A = 240x - 32x^2$		A1 N2	
					[2]
	(e)	The x-coordinate of the vertex of the graph of $y = 240x - 32x^2$			
		$= -\dfrac{240}{2(-32)}$		A1	
		$= 3.75$			
		$\neq 3.3333342$			
		Therefore, the total surface area of the box does not attain its maximum when its volume attains its maximum.		R1	
		Thus, the claim is incorrect.		AG N0	
					[2]

5. (a) 2 A1 N1

[1]

(b) $f(3) = \frac{4}{3}(3)^3 + 5(3)^2 - 6(3) + 2$ (M1) for substitution

$f(3) = 65$ A1 N2

[2]

(c) $f'(x) = \frac{4}{3}(3x^2) + 5(2x) - 6(1) + 0$ (A1) for correct derivatives

$f'(x) = 4x^2 + 10x - 6$ A1 N2

[2]

(d) $4x^2 + 10x - 6 = 0$

$2(x+3)(2x-1) = 0$ (M1) for valid approach

$x = -3$ or $x = \frac{1}{2}$ A2 N3

[3]

(e) $y = 29$, $y = \frac{5}{12}$ A2 N2

[2]

(f) (i) $\frac{5}{12} < w < 29$ A2 N2

(ii) $w < \frac{5}{12}$ or $w > 29$ A2 N2

[4]

(g) The gradient of the tangent

$= f'(3)$

$= 4(3)^2 + 10(3) - 6$ (A1) for substitution

$= 60$ A1 N2

[2]

(h) The equation of the normal:

$y - 65 = \frac{-1}{60}(x-3)$ M1A1

$-60y + 3900 = x - 3$ A1

$x + 60y - 3903 = 0$ AG N0

[3]

AI SL Practice Set 2 Paper 1 Solution

1. (a) (i) 40 A1 N1

(ii) 1 A1 N1

(iii) 0 A1 N1

[3]

(b) The mean number of watermelons

$$=\frac{(0)(12)+(1)(10)+(2)(6)+(3)(5)+(4)(5)+(5)(2)}{12+10+6+5+5+2}$$ (A1) for correct formula

$=1.675$ A1 N2

[2]

(c) Discrete A1 N1

[1]

2. (a) The required perimeter

$=120+350+370$ (M1) for valid approach

$=840\text{ cm}$

$=8.4\times10^{2}\text{ cm}$ A1 N2

[2]

(b) The required area

$$=\frac{(120)(350)}{2}$$ (M1) for valid approach

$=21000\text{ cm}^{2}$

$=2.1\times10^{4}\text{ cm}^{2}$ A1 N2

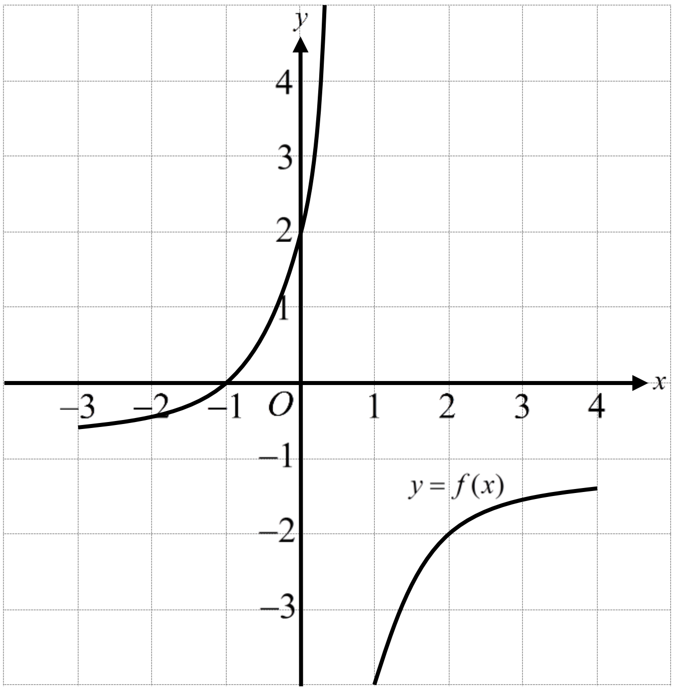
[2]

3. (a) For correct asymptotic behavior at $x=\frac{1}{2}$ A1

For correct intercepts A1

For correct shape A1 N3

[3]



(b) (i) $x=\frac{1}{2}$ A1 N1

(ii) -1 A1 N1

[2]

4. (a) Let h m be the height of the tower.

$\tan 21.7^{\circ}=\frac{h}{1.5}$ (M1) for valid approach

$h=0.5969224984$ (A1) for correct value

Thus, the height of the tower is 597 m. A1 N3

[3]

(b) The percentage error

$=\left|\frac{596.9224984-603}{603}\right|\times 100\%$ (A1) for substitution

$=1.007877552\%$

$=1.01\%$ A1 N2

[2]

5. (a) (i) x_n A1 N1

(ii) z_n A1 N1

[2]

(b) The required term

$= 100 + (10 - 1)(200)$ (A1) for substitution

$= 1900$ A1 N2

[2]

(c) The required sum

$= \dfrac{100(3^{10} - 1)}{3 - 1}$ (A1) for substitution

$= 2952400$ A1 N2

[2]

6. (a) (i) 3.5 A1 N1

(ii) 9.5 A1 N1

(iii) 2.5 A1 N1

[3]

(b) The period of d

$= \dfrac{360^\circ}{3^\circ}$ (M1) for valid approach

$= 120 \text{ minutes}$ A1 N2

[2]

(c) $10:30 \text{ am}$ A1 N1

[1]

7. (a) $x + y = 2000$ A1 N1

[1]

(b) (i) $50x + 15y = 73750$ A1 N1

(ii) $x = 1250$ A1 N1

$y = 750$ A1 N1

[3]

(c) The total cost

$= 50(2) + 15(12)$ (M1) for substitution

$= \$280$ A1 N2

[2]

8. (a) 16500 A1 N1

[1]

(b) The number of followers

$=16500(1.07)^{17}$ (M1) for substitution

$=52120.45098$

$=52120$ A1 N2

[2]

(c) $N(t)=500000$

$16500(1.07)^t=500000$ (M1) for setting equation

$16500(1.07)^t-500000=0$

By considering the graph of

$y=16500(1.07)^t-500000$, $t=50.418502$. (A1) for correct value

Thus, the corresponding year is 2023. A1 N3

[3]

9. (a) (i) The required radius

$=\sqrt{(12-8)^2+(14-11)^2}$ (A1) for substitution

$=5$ A1 N2

(ii) The required radius

$=\sqrt{\left(6-\frac{41}{7}\right)^2+\left(2-\frac{57}{7}\right)^2}$ (A1) for substitution

$=6.144518048$

$=6.14$ A1 N2

[4]

(b) F A1 N1

[1]

10. (a) By TVM Solver:

$N = ?$
$I\% = 2.95$
$PV = 120000$
$PMT = -2000$
$FV = 0$
$P/Y = 12$
$C/Y = 12$
$PMT : END$

(M1)(A1) for correct values

$N = 64.99449865$

Thus, the number of months to repay the loan is 65 months. A1 N3

[3]

(b) The amount of interest paid

$= (2000)(65) - 120000$ (M1)(A1) for substitution

$= \$10000$ A1 N3

[3]

11. (a) $E(X) = (54)(0.07)$ (A1) for substitution

$E(X) = 3.78$ A1 N2

[2]

(b) $P(X = 9)$

$= 0.0081914007$ (A1) for correct value

$= 0.00819$ A1 N2

[2]

(c) $P(X \geq 5)$

$= 1 - P(X \leq 4)$ (M1) for valid approach

$= 1 - 0.6733974584$ (A1) for correct value

$= 0.3266025416$

$= 0.327$ A1 N3

[3]

12. (a) The required cost

$=\frac{1}{2}(100-90)^2+60$ (M1) for substitution

$=\$110$ A1 N2

[2]

(b) $C(x)\le 1310$

$\frac{1}{2}(x-90)^2+60\le 1310$ (M1) for setting inequality

$\frac{1}{2}(x-90)^2-1250\le 0$

By considering the graph of

$y=\frac{1}{2}(x-90)^2-1250$, $40\le x\le 140$.

$\therefore n=40$ A1 N2

[2]

(c) The minimum point of the graph of $C(x)$ is

$(90, 60)$. (M1) for valid approach

Thus, the required number of jackets is 90. A1 N2

[2]

13. (a) $f(x)=\int\left(\frac{1000}{x^2}+500x\right)\mathrm{d}x$ (M1) for indefinite integral

$f(x)=1000\left(\frac{x^{-1}}{-1}\right)+500\left(\frac{x^2}{2}\right)+C$ (A1) for correct approach

$f(x)=-\frac{1000}{x}+250x^2+C$ (A1) for correct approach

$600=-\frac{1000}{2}+250(2)^2+C$ (M1) for substitution

$600=500+C$

$C=100$

$\therefore f(x)=-\frac{1000}{x}+250x^2+100$ A1 N5

[5]

(b) $q=5$ A1 N1

[1]

14. (a) (i) 0.683 A1 N1

(ii) 0.954 A1 N1

[2]

(b) $\text{P}(H < 2.82)$

$= 0.4372698598$ (A1) for correct value

$= 0.437$ A1 N2

[2]

(c) $\text{P}(H > r) = 0.28$ (M1) for valid approach

$\text{P}(H < r) = 0.72$

$r = 2.960739885$

$r = 2.96$ A1 N2

[2]

AI SL Practice Set 2 Paper 2 Solution

1.	(a)	(i)	$\bar{x} = 30000$	A1	N1
		(ii)	$\bar{y} = 9980$	A1	N1
		(iii)	$a = -0.176$	A1	N1
		(iv)	$b = 15260$	A1	N1
		(v)	$r = -0.9809315165$	(A1) for correct value	
			$r = -0.981$	A1	N2

[6]

(b)	The estimated insurance cost			
	$= -0.176(32500) + 15260$	(A1) for substitution		
	$= \$9540$	A1	N2	

[2]

(c)	The data 52500 km is outside the range of values of x.	R1	N1

[1]

(d)	(i)	H_0: The insurance cost follows the assigned distribution.	A1	N1
	(ii)	$p\text{-value} = 0.1031478315$	(A1) for correct value	
		$p\text{-value} = 0.103$	A1	N2
	(iii)	The null hypothesis is not rejected.	A1	
		As $p\text{-value} > 0.05$.	R1	N2

[5]

2. (a) $7(98)+24f-2990=0$ (M1) for setting equation

$24f=2304$

$f=96$ A1 N2

[2]

(b) $-\dfrac{7}{24}$ A1 N1

[1]

(c) (i) The gradient of DE

$=-1\div-\dfrac{7}{24}$ (M1) for valid approach

$=\dfrac{24}{7}$ A1 N2

(ii) The equation of DE:

$y-10=\dfrac{24}{7}(x-125)$ M1A1

$7y-70=24(x-125)$ A1

$7y-70=24x-3000$

$24x-7y-2930=0$ AG N0

[5]

(d) $(146, 82)$ A2 N2

[2]

(e) The coordinates of the mid-point of CD

$=\left(\dfrac{50+146}{2}, \dfrac{110+82}{2}\right)$ M1A1

$=(98, 96)$

Thus, F is the mid-point of CD. AG N0

[2]

(f) The length of DE

$=\sqrt{(146-125)^2+(82-10)^2}$ (A1) for substitution

$=75$ A1 N2

[2]

(g) The area of the triangle CDE

$=\dfrac{(75)(100)}{2}$ (M1) for valid approach

$=3750\text{ m}^2$ A1 N2

[2]

(h) The total area

$= 3750 + \frac{(BC + AE)(AB)}{2}$ (M1)(A1) for correct approach

$= 3750 + \frac{(40 + 115)(100)}{2}$ (A1) for substitution

$= 11500 \text{ m}^2$ A1 N4

[4]

3. (a) $H_1: \mu_1 > \mu_2$ A1 N1

[1]

(b) $p\text{-value} = 0.0231895114$ (A1) for correct value

$p\text{-value} = 0.0232$ A1 N2

[2]

(c) The null hypothesis is rejected. A1

As $p\text{-value} < 0.05$. R1 N2

[2]

(d) (i) The required probability

$= \left(\frac{5}{10}\right)\left(\frac{2}{9}\right)$ (A1) for correct formula

$= \frac{1}{9}$ A1 N2

(ii) The required probability

$= \left(\frac{5}{10}\right)\left(\frac{2}{9}\right) + \left(\frac{5}{10}\right)\left(\frac{7}{9}\right) + \left(\frac{5}{10}\right)\left(\frac{2}{9}\right)$ (A1) for correct formula

$= \frac{11}{18}$ A1 N2

[4]

(e) H_1: The age and the reading preference are not independent. A1 N1

[1]

(f) 4 A1 N1

[1]

(g) $\chi^2_{calc} = 53.64204545$ (A1) for correct value

$\chi^2_{calc} = 53.6$ A1 N2

[2]

(h) The null hypothesis is rejected. A1

As $\chi^2_{calc} > 13.277$. R1 N2

[2]

4. (a) $AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos A\hat{B}C$ (M1) for cosine rule

$AC^2 = 15^2 + 13.5^2 - 2(15)(13.5)\cos 98^\circ$ (A1) for substitution

$AC = 21.53172324\text{ m}$

$AC = 21.5\text{ m}$ A1 N3

[3]

(b) $\dfrac{\sin B\hat{A}C}{BC} = \dfrac{\sin A\hat{B}C}{AC}$ (M1) for sine rule

$\dfrac{\sin B\hat{A}C}{13.5} = \dfrac{\sin 98^\circ}{21.53172324}$ (A1) for substitution

$\sin B\hat{A}C = \dfrac{13.5\sin 98^\circ}{21.53172324}$

$B\hat{A}C = 38.38043409^\circ$

$B\hat{A}C = 38.4^\circ$ A1 N3

[3]

(c) The area of the triangular region ABC

$= \dfrac{1}{2}(AB)(BC)\sin A\hat{B}C$ (M1) for area formula

$= \dfrac{1}{2}(15)(13.5)\sin 98^\circ$ (A1) for substitution

$= 100.264642\text{ m}^2$

$= 100\text{ m}^2$ A1 N3

[3]

(d) The height of the vertical pole VA

$= 15\tan 22.1^\circ$ (M1) for valid approach

$= 6.090868387\text{ m}$ (A1) for correct value

Let θ be the required angle of depression.

$\tan\theta = \dfrac{6.090868387}{21.53172324}$ (M1) for valid approach

$\theta = 15.79508441^\circ$

Thus, the angle of depression of C from V is 15.8°. A1 N4

[4]

5.	(a)	$f'(x) = -3x^2 + b(2x) - 432(1) + 0$	(A1) for correct derivatives		
		$f'(x) = -3x^2 + 2bx - 432$			
		$f'(8) = 0$	(M1) for setting equation		
		$\therefore -3(8)^2 + 2b(8) - 432 = 0$	(A1) for substitution		
		$16b = 624$			
		$b = 39$	A1	N4	
					[4]
	(b)	(i) 984	A1	N1	
		(ii) $(18, 1484)$	A2	N2	
					[3]
	(c)	$8 < x < 18$	A2	N2	
					[2]
	(d)	(i) $984 < k < 1484$	A2	N2	
		(ii) $k \le 984$ or $k \ge 1484$	A2	N2	
					[4]
	(e)	$C(x) = -x^3 + 39x^2 - 432x + 2456$			
		$C(8) = 984$			
		$C(25)$			
		$= -25^3 + 39(25)^2 - 432(25) + 2456$	A1		
		$= 406$			
		$C(8) > C(25)$	R1		
		Thus, the average cost attains its minimum when 25000 smart watches are produced.	AG	N0	
					[2]
	(f)	$C(x) \le 984$	(M1) for setting inequality		
		$-x^3 + 39x^2 - 432x + 2456 \le 984$			
		$-x^3 + 39x^2 - 432x + 1472 \le 0$			
		By considering the graph of $y = -x^3 + 39x^2 - 432x + 1472$, $x = 8$ or $x \ge 23$.			
		Thus, the range of values of x are $x = 8$ or $23 \le x \le 25$.	A2	N3	
					[3]

AI SL Practice Set 3 Paper 1 Solution

1. (a) $\$60300000$ A1 N1

[1]

(b) $\$6.03\times10^{7}$ A2 N2

[2]

(c) The percentage error

$$=\left|\frac{60300000-61204500}{61204500}\right|\times100\%$$ (A1) for substitution

$$=1.477832512\%$$

$$=1.48\%$$ A1 N2

[2]

2. (a) The coordinates of the mid-point

$$=\left(\frac{3+9}{2},\frac{5+7}{2}\right)$$ (A1) for substitution

$$=(6,6)$$ A1 N2

[2]

(b) The gradient of L

$$=\frac{7-5}{9-3}$$ (A1) for substitution

$$=\frac{1}{3}$$ A1 N2

[2]

(c) The equation of L:

$$y-5=\frac{1}{3}(x-3)$$ (A1) for substitution

$$y-5=\frac{1}{3}x-1$$

$$y=\frac{1}{3}x+4$$ A1 N2

[2]

3. (a) $260-100=(31-11)d$ (M1) for valid approach

$160=20d$

$d=8$

Thus, the common difference is 8. A1 N2

[2]

(b) $u_{11}=100$

$\therefore u_1+(11-1)(8)=100$ (A1) for correct equation

$u_1=20$ A1 N2

[2]

(c) S_{51}

$=\frac{51}{2}\left[2(20)+(51-1)(8)\right]$ (A1) for substitution

$=11220$ A1 N2

[2]

4. (a) 4 A1 N1

[1]

(b) The inter-quartile range

$=6-2.5$ (M1) for valid approach

$=3.5$ A1 N2

[2]

(c) The required probability

$=\frac{8}{12}$ (M1) for valid approach

$=\frac{2}{3}$ A1 N2

[2]

5. (a) The common ratio

$$=\sqrt{\frac{20}{9}\div 20}$$ (M1) for valid approach

$$=\frac{1}{3}$$ A1 N2

[2]

(b) $\frac{20}{81}$ A1 N1

[1]

(c) $S_n=\frac{65600}{2187}$

$$\therefore \frac{20\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}}=\frac{65600}{2187}$$ (A1) for correct equation

$$30\left(1-\left(\frac{1}{3}\right)^n\right)-\frac{65600}{2187}=0$$ (A1) for correct approach

By considering the graph of

$y=30\left(1-\left(\frac{1}{3}\right)^n\right)-\frac{65600}{2187}$, $n=8$. A1 N3

[3]

6. (a) $\mathrm{P}(X=1)+\mathrm{P}(X=2)+\mathrm{P}(X=3)+\mathrm{P}(X=4)=1$ M1

$\therefore 5k^2+(k^2+6k)+(k^2+k)+k^2=1$ A1

$8k^2+7k-1=0$

$(k+1)(8k-1)=0$ A1

$k=-1$ (*Rejected*) or $k=\frac{1}{8}$ AG N0

[3]

(b) $\mathrm{P}(X=2\,|\,X\le 2)$

$=\frac{\mathrm{P}(X=2\cap X\le 2)}{\mathrm{P}(X\le 2)}$

$=\frac{\mathrm{P}(X=2)}{\mathrm{P}(X\le 2)}$ (M1) for valid approach

$=\frac{\left(\frac{1}{8}\right)^2+6\left(\frac{1}{8}\right)}{5\left(\frac{1}{8}\right)^2+\left(\left(\frac{1}{8}\right)^2+6\left(\frac{1}{8}\right)\right)}$ (A1) for substitution

$=\frac{49}{54}$ A1 N3

[3]

7. (a) (i) $\begin{cases}15a+7b+2c=97\\3a+5b+9c=99\\4a+4c=48\end{cases}$ A2 N2

(ii) $a=4$, $b=3$ and $c=8$ A3 N3

[5]

(b) $248 A1 N1

[1]

8. (a) $h = -\frac{b}{2a}$

$\therefore -5 = -\frac{10}{2a}$ (A1) for correct equation

$-5 = -\frac{5}{a}$

$a = 1$ A1 N2

[2]

(b) $0 = (-8)^2 + 10(-8) + c$ (M1) for setting equation

$c = 16$ A1 N2

[2]

(c) $\{y : y \geq -9, y \in \mathbb{R}\}$ A1 N1

[1]

9. (a) $\cos A\hat{C}B = \frac{AC^2 + BC^2 - AB^2}{2(AC)(BC)}$ (M1) for cosine rule

$\cos A\hat{C}B = \frac{54^2 + 54^2 - 35^2}{2(54)(54)}$ (A1) for substitution

$\cos A\hat{C}B = 0.789951989$

$A\hat{C}B = 37.81897498^\circ$

$A\hat{C}B = 37.8^\circ$ A1 N3

[3]

(b) The required area

$= \frac{1}{2}(AC)(BC)\sin A\hat{C}B$ (M1) for area formula

$= \frac{1}{2}(54)(54)\sin 37.81897498^\circ$ (A1) for substitution

$= 893.999965 \text{ cm}^2$

$= 894 \text{ cm}^2$ A1 N3

[3]

10. (a) $\frac{\mathrm{d}y}{\mathrm{d}x}$

$=\frac{1}{4}(4x^3)+2(2x)+0$ (A1) for correct derivatives

$=x^3+4x$ A1 N2

[2]

(b) The gradient of the tangent at Q

$=2^3+4(2)$ (M1) for substitution

$=16$ A1 N2

[2]

(c) The equation of the tangent at Q:

$y-15=16(x-2)$ (M1) for substitution

$y-15=16x-32$

$16x-y-17=0$ A1 N2

[2]

11. (a) $y=5$ A1 N1

[1]

(b) (i) $\left(5,\frac{7}{2}\right)$ A1 N1

(ii) $k(5)+2\left(\frac{7}{2}\right)-47=0$ (M1) for substitution

$5k=40$

$k=8$ A1 N2

(iii) $8x+2(5)-47=0$ (M1) for substitution

$8x=37$

$x=\frac{37}{8}$

Thus, the required coordinates are

$\left(\frac{37}{8},5\right)$. A1 N2

[5]

12. (a) $y=\frac{8}{7}$ A2 N2

[2]

(c) $\left\{y: y\neq\frac{8}{7}, y\in\mathbb{R}\right\}$ A1 N1

[1]

(d) $f(x)>g(x)$

$\frac{1-8x}{2-7x}>\frac{1}{2}x^2$

$\frac{1-8x}{2-7x}-\frac{1}{2}x^2>0$ M1

By considering the graph of $y=\frac{1-8x}{2-7x}-\frac{1}{2}x^2$,

$-1.439727<x<0.1239131$ or $\frac{2}{7}<x<1.6015283$.

$\therefore -1.44<x<0.124$ or $\frac{2}{7}<x<1.60$ A2 N3

[3]

13. (a) Let $r\%$ be the nominal annual interest rate compounded yearly.

$(1+r\%)^6=\left(1+\frac{9}{(100)(12)}\right)^{(12)(6)}$ (A1) for substitution

$1+r\%=1.0075^{12}$

$r=9.380689767$ (A1) for correct value

The real interest rate per year

$=9.380689767\%-i\%$

$=(9.38069-i)\%$ A1 N3

[3]

(b) $89000\left(1+\frac{9.38069-i}{100}\right)^6=118000$ (M1) for setting equation

$89000\left(1+\frac{9.38069-i}{100}\right)^6-118000=0$ (A1) for correct approach

By considering the graph of

$y=89000\left(1+\frac{9.38069-i}{100}\right)^6-118000$,

$i=4.5676461$.

Thus, $i=4.57$. A1 N3

[3]

14.	(a)	0.0707	A1	N1
				[1]
	(b)	$\mathrm{P}(H > q) = 0.37$	(M1) for valid approach	
		$\mathrm{P}(H < q) = 0.63$		
		$q = 6.225660279$		
		$q = 6.23$	A1	N2
				[2]
	(c)	$\mathrm{P}(6 - t < H < 6 + t) = 0.8$	(M1) for valid approach	
		$\mathrm{P}(H < 6 - t) = 0.1$		
		$6 - t = 5.128544935$		
		$t = 0.8714550653$		
		$t = 0.871$	A1	N2
				[2]

AI SL Practice Set 3 Paper 2 Solution

1. (a) $a = 5.6$ A1 N1

$b = 34.8$ A1 N1

[2]

(b) The estimated hardness

$= 5.6(6.3) + 34.8$ (A1) for substitution

$= 70.08$ A1 N2

[2]

(c) The required probability

$= \frac{120 - 56}{120}$ (M1) for valid approach

$= \frac{8}{15}$ A1 N2

[2]

(d) (i) Let X be the number of selected ingots of the hardness at least 65, where $X \sim \mathrm{B}\left(10, \frac{8}{15}\right)$.

The required probability

$= \mathrm{P}(X = 5)$ (M1) for valid approach

$= 0.2406733955$

$= 0.241$ A1 N2

(ii) The required probability

$= \mathrm{P}(X < 4)$ (M1) for valid approach

$= 0.1226252054$

$= 0.123$ A1 N2

(iii) $\frac{16}{3}$ A1 N1

[5]

(d) (i) $H_1 : \mu_1 \neq \mu_2$ A1 N1

(ii) $p\text{-value} = 0.0741679182$ (A1) for correct value

$p\text{-value} = 0.0742$ A1 N2

(iii) The null hypothesis is not rejected. A1

As $p\text{-value} > 0.05$. R1 N2

[5]

2. (a) The volume

$= \pi r^2 h$

$= \pi(4)^2(15)$ (A1) for substitution

$= 240\pi \text{ cm}^3$ A1 N2

[2]

(b) The total surface area

$= 2\pi r^2 + 2\pi rh$

$= 2\pi(4)^2 + 2\pi(4)(15)$ (A1) for substitution

$= 152\pi \text{ cm}^2$ A1 N2

[2]

(c) 26 A1 N1

[1]

(d) $l^2 h = \pi r^2 h$ (M1) for setting equation

$l^2 = \pi r^2$

$\therefore l^2 = \pi(4)^2$ (A1) for substitution

$l = \sqrt{16\pi}$

$l = 7.089815404 \text{ cm}$

$l = 7.09 \text{ cm}$ A1 N3

[3]

(e) The total surface area of the new container

$= 2l^2 + 4lh$ M1

$= 2(7.089815404)^2 + 4(7.089815404)(15)$ A1

$= 525.9198891 \text{ cm}^2$

$> 152\pi \text{ cm}^2$ R1

Thus, the claim is agreed. A1 N0

[4]

3. (a) (i) H_0: The punctuality of buses and the locations of bus stops are independent. A1 N1

(ii) H_1: The punctuality of buses and the locations of bus stops are not independent. A1 N1

[2]

(b) 8 A1 N1

[1]

(c) $\chi^2_{calc} = 19.37210492$ (A1) for correct value

$\chi^2_{calc} = 19.4$ A1 N2

[2]

(d) The null hypothesis is rejected. A1

As $\chi^2_{calc} > 15.507$. R1 N2

[2]

(e) (i) The required probability

$= \frac{48}{500}$ (A1) for correct formula

$= \frac{12}{125}$ A1 N2

(ii) The required probability

$= \frac{15+13+8+11+8}{500}$ (A1) for correct formula

$= \frac{11}{100}$ A1 N2

(iii) The required probability

$= \frac{11}{15+13+8+11+8}$ (A1) for correct formula

$= \frac{1}{5}$ A1 N2

[6]

(f) The required probability

$= \left(\frac{74}{500}\right)\left(\frac{74-1}{500-1}\right)\left(\frac{74-2}{500-2}\right)$ (A2) for correct formula

$= 0.0031303088$

$= 0.00313$ A1 N3

[3]

4. (a) $P(0)=116$

$\therefore a+b\times c^{0}=116$ (M1) for setting equation

$a+b=116$ A1 N2

[2]

(b) $P(1)=172$

$\therefore a+b\times c^{-1}=172$ (M1) for setting equation

$a+\frac{b}{c}=172$ A1 N2

[2]

(c) (i) $\log_{c}81=4$

$\therefore c^{4}=81$ M1

$c^{4}=3^{4}$ A1

$c=3$ AG N0

(ii) The system is $\begin{cases} a+b=116 \\ a+\frac{1}{3}b=172 \end{cases}$. (M1) for valid approach

Solving, we have $a=200$ and $b=-84$. A2 N3

[5]

(d) The number of elephants

$=200-84\times3^{-3}$ (M1) for substitution

$=196.8888889$

$=197$ A1 N2

[2]

(e) 200 A1 N1

[1]

(f) $200-84\times3^{-t}>195$ (M1) for setting inequality

$5-84\times3^{-t}>0$

By considering the graph of $y=5-84\times3^{-t}$,

$t=2.5681297$.

Thus, the number of years needed is 2.57 years. A1 N2

[2]

(g) By considering the graphs of $y = 200 - 84 \times 3^{-t}$, $y = 170$, $y = 180$ and $y = 190$, y reaches 170, 180 and 190 at $t_1 = 0.9372$, $t_2 = 1.3062702$ and $t_3 = 1.9372$ respectively. M1A1

$\because 2(t_2 - t_1)$

$= 2(1.3062702 - 0.9372)$

$= 0.7381404$

$\neq t_3 - t_2$ R1

Thus, the claim is disagreed. A1 N0

[4]

5.	(a)	(i)	$(4, 8)$	A2	N2
		(ii)	$\{y : 4 \le y \le 8,\ y \in \mathbb{R}\}$	A2	N2
					[4]
	(b)		$f'(x)$		
			$= -0.25(2x) + 2(1) + 0$	(A1) for correct derivatives	
			$= -0.5x + 2$	A1	N2
					[2]
	(c)		$f'(x) = -1$	M1	
			$\therefore -0.5x + 2 = -1$	A1	
			$-0.5x = -3$		
			$x = 6$	A1	
			$f(6)$		
			$= -0.25(6)^2 + 2(6) + 4$	M1	
			$= 7$		
			Thus, the coordinates of P are $(6, 7)$.	AG	N0
					[4]
	(d)		The equation of the tangent:		
			$y - 7 = -1(x - 6)$	(A1) for substitution	
			$y - 7 = -x + 6$		
			$x + y - 13 = 0$	A1	N2
					[2]
	(e)	(i)	4	A1	N1
		(ii)	5.75	A1	N1
					[2]
	(f)		The estimate of $\int_0^8 f(x)\mathrm{d}x$		
			$= \frac{1}{2}(1)\left[4 + 4 + 2\begin{pmatrix} 5.75 + 7 + 7.75 \\ +8 + 7.75 + 7 + 5.75 \end{pmatrix}\right]$	(A2) for substitution	
			$= 53$	A1	N3
					[3]
	(g)		Underestimate	A1	N1
					[1]

AI SL Practice Set 4 Paper 1 Solution

1. (a) (i) The distance travelled

$= 2\pi(1425000000)$ (M1) for valid approach

$= 8953539063 \text{ km}$

$= 8950000000 \text{ km}$ A1 N2

(ii) The distance travelled

$= \dfrac{2\pi(1425000000)}{(29)(365)}$ (M1) for valid approach

$= 845870.483 \text{ km}$

$= 846000 \text{ km}$ A1 N2

[4]

(b) $8.46 \times 10^5 \text{ km}$ A2 N2

[2]

2. (a) $V = \dfrac{1}{3}\pi r^2 h$

$\therefore 128\pi = \dfrac{1}{3}\pi r^2(6)$ (A1) for correct equation

$r^2 = 64$

$r = 8$

Thus, the required radius is 8 cm. A1 N2

[2]

(b) l

$= \sqrt{r^2 + h^2}$ (M1) for valid approach

$= \sqrt{8^2 + 6^2}$

$= 10$

Thus, the required slant height is 10 cm. A1 N2

[2]

(c) The total surface area

$= \pi r^2 + \pi r l$

$= \pi(8)^2 + \pi(8)(10)$ (A1) for substitution

$= 144\pi \text{ cm}^2$ A1 N2

[2]

3. (a) (i) $-\frac{1}{26}$ A1 N1

(ii) -0.038462 A1 N1

[2]

(b) The percentage error

$=\left|\frac{-0.039-(-0.038462)}{-0.038462}\right|\times 100\%$ (A1) for substitution

$=1.398783215\%$

$=1.40\%$ A1 N2

[2]

4. (a) (i) $\begin{cases} 7x+8y+5z=49 \\ 4x+6y+10z=18 \\ 11x+9y=82 \end{cases}$ A2 N2

(ii) $x=5$, $y=3$ and $z=-2$ A3 N3

[5]

(b) A team drops two points for losing a game. A1 N1

[1]

5. (a) (i) 20 hours A1 N1

(ii) 15 hours A1 N1

[2]

(b) 5 workers worked for more than 30 hours. Therefore, 12.5% of the workers worked for more than 30 hours. (R1) for correct argument

$\therefore k=30$ A1 N2

[2]

6.	(a)	(i)	$\{0, 1, 2, 3, 4, 5\}$	A1	N1	
		(ii)	$\{-1, 1, 11, 35, 79, 149\}$	A2	N2	
						[3]
	(b)		$g(x) = h(x)$ $x^3 + x^2 - 1 = 98\ln(0.57x)$ $x^3 + x^2 - 1 - 98\ln(0.57x) = 0$ By considering the graph of $y = x^3 + x^2 - 1 - 98\ln(0.57x)$, $x = 1.9459391$ or $x = 4.0546399$. $\therefore x = 1.95$ or $x = 4.05$	A2	N2	
						[2]
7.	(a)		H_0: The outcomes follows the assigned distribution.	A1	N1	
						[1]
	(b)		50	A1	N1	
						[1]
	(c)		4	A1	N1	
						[1]
	(d)		$p\text{-value} = 0.0003344965427$	(A1) for correct value		
			$p\text{-value} = 0.000334$	A1	N2	
						[2]
	(e)		The null hypothesis is rejected.	A1		
			As $p\text{-value} < 0.05$.	R1	N2	
						[2]

8.	(a)	(i)	c_n	A1	N1	
		(ii)	b_n	A1	N1	
						[2]
	(b)	(i)	1.25	A1	N1	
		(ii)	$\frac{3125}{128}$	A1	N1	
		(iii)	S_8			
			$=\frac{10(1.25^8-1)}{1.25-1}$	(A1) for substitution		
			$=198.4185791$			
			$=198$	A1	N2	
						[4]
9.	(a)	(i)	The radius			
			$=\sqrt{(10-6)^2+(12-14)^2}$	(A1) for substitution		
			$=4.472135955\text{ km}$			
			$=4.47$ km	A1	N2	
		(ii)	4 km	A1	N1	
		(iii)	The apartment at P	A1	N1	
						[4]
	(b)		$x+y-20=0$	A2	N2	
						[2]

10. (a) The initial number of tigers. A1 N1

[1]

(b) 500 A1 N1

[1]

(c) The required number

$= P(7)$

$= \frac{500}{\ln 2}(\ln(7+2))$ (M1) for substitution

$= 1584.962501$

$= 1580$ A1 N2

[2]

(d) $P(t) = 1600$

$\therefore \frac{500}{\ln 2}(\ln(t+2)) = 1600$ (M1) for setting equation

$\frac{500}{\ln 2}(\ln(t+2)) - 1600 = 0$

By considering the graph of

$y = \frac{500}{\ln 2}(\ln(t+2)) - 1600$, $t = 7.1895868$.

Thus, the number of complete days needed is 8. A1 N2

[2]

11. (a) $\mathrm{E}(X) = 8.64$

$\therefore 0.72n = 8.64$ (A1) for correct equation

$n = 12$ A1 N2

[2]

(b) $\mathrm{Var}(X)$

$= (12)(0.72)(1-0.72)$ (A1) for substitution

$= 2.4192$ A1 N2

[2]

(c) $\mathrm{P}(X \geq 11)$

$= 1 - \mathrm{P}(X \leq 10)$ (A1) for substitution

$= 0.1099809898$

$= 0.110$ A1 N2

[2]

12. (a) By TVM Solver:

$$\boxed{\begin{array}{l} N=120 \\ I\%=4.5 \\ PV=0 \\ PMT=-200 \\ FV=? \\ P/Y=12 \\ C/Y=1 \\ PMT:END \end{array}}$$

(A2) for correct values

$FV=30095.13482$

Thus, the value of the investment after ten years is $\$30100$. A1 N3

[3]

(b) By TVM Solver:

$$\boxed{\begin{array}{l} N=144 \\ I\%=4.5 \\ PV=0 \\ PMT=? \\ FV=5\times 30095.13482 \\ P/Y=12 \\ C/Y=1 \\ PMT:END \end{array}}$$

(A2) for correct values

$PMT=-794.6316652$

Thus, the new amount of deposit is $\$795$. A1 N3

[3]

13. (a) x

$= -\frac{b}{2a}$

$= -\frac{100}{2(-1)}$ (A1) for substitution

$= 50$ A1 N2

[2]

(b) The required maximum height

$= -50^2 + 100(50) - 1600$ A1

$= -2500 + 5000 - 1600$

$= 900 \text{ m}$ AG N0

[1]

(c) $V = 0$

$-x^2 + 100x - 1600 = 0$

$x = 20$ or $x = 80$ (A1) for correct values

The required horizontal distance

$= 80 - 20$ (M1) for valid approach

$= 60 \text{ m}$ A1 N3

[3]

14. (a) $P'(x) = 3x^2 - 135(2x) + 5400(1)$ (A1) for correct derivatives

$P'(x) = 3x^2 - 270x + 5400$ A1 N2

[2]

(b) $P'(x) = 0$

$3x^2 - 270x + 5400 = 0$ (M1) for setting equation

By considering the graph of

$y = 3x^2 - 270x + 5400$, $x = 30$ or

$x = 60$ (*Rejected*). (M1) for valid approach

Thus, the required number of loudspeakers is 30. A1 N3

[3]

(c) $67500 A1 N1

[1]

AI SL Practice Set 4 Paper 2 Solution

1. (a) The gradient of L_1

$= \frac{40-0}{0-30}$ (A1) for substitution

$= -\frac{4}{3}$ A1 N2

[2]

(b) The equation of L_1:

$y-40 = -\frac{4}{3}(x-0)$ (A1) for substitution

$3y-120 = -4x$

$4x+3y-120 = 0$ A1 N2

[2]

(c) The gradient of L_2

$= -1 \div -\frac{4}{3}$

$= \frac{3}{4}$ (A1) for correct value

The equation of L_2:

$y = \frac{3}{4}x$ A1 N2

[2]

(d) $4x+3\left(\frac{3}{4}x\right)-120 = 0$ (M1) for substitution

$6.25x = 120$

$x = 19.2$

$y = \frac{3}{4}(19.2)$ (M1) for substitution

$y = 14.4$

Thus, the coordinates of C are $(19.2, 14.4)$. A1 N3

[3]

(e) The area of the triangle OBC

$= \frac{(40-0)(19.2-0)}{2}$ (M1) for valid approach

$= 384$ A1 N2

[2]

(f) $\text{BC} = \sqrt{(0-19.2)^2 + (40-14.4)^2}$ (A1) for substitution

$\text{BC} = 32$ (A1) for correct value

$\text{OC} = \sqrt{(19.2-0)^2 + (14.4-0)^2}$

$\text{OC} = 24$ (A1) for correct value

The perimeter of the triangle OBC

$= 24 + 32 + 40$

$= 96$ A1 N4

[4]

(g) $\frac{3}{4}k$ A1 N1

[1]

(h) $\frac{(\text{BC})(\text{CD})}{2} = 624$ (A1) for correct equation

$32\text{CD} = 1248$

$\text{CD} = 39$ (A1) for correct value

$\therefore \sqrt{(k-19.2)^2 + \left(\frac{3}{4}k - 14.4\right)^2} = 39$ (A1) for correct equation

$\sqrt{(k-19.2)^2 + \left(\frac{3}{4}k - 14.4\right)^2} - 39 = 0$

By considering the graph of

$y = \sqrt{(k-19.2)^2 + \left(\frac{3}{4}k - 14.4\right)^2} - 39$, $k = -12$ or

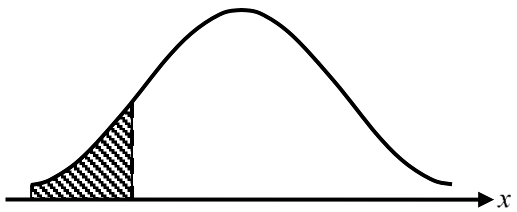
$k = 50.4$ (*Rejected*).

$\therefore k = -12$ A1 N4

[4]

2. (a) For vertical line clearly to the left of the mean A1
For shading to the left of the vertical line A1 N2

[2]



(b) (i) Let X be the volume of a randomly selected milk soda.
The required probability
$= \mathrm{P}(X < 490)$ (M1) for valid approach
$= 0.105649839$
$= 0.106$ A1 N2

(ii) The required probability
$= \mathrm{P}(X > 483 \mid X < 490)$ (M1) for valid approach
$= \dfrac{\mathrm{P}(X > 483 \cap X < 490)}{\mathrm{P}(X < 490)}$
$= \dfrac{\mathrm{P}(483 < X < 490)}{\mathrm{P}(X < 490)}$ (A1) for correct approach
$= 0.8410480651$
$= 0.841$ A1 N3

[5]

(c) The required probability
$= 2 \times \mathrm{P}(X < 490) \times (1 - \mathrm{P}(X < 490))$ (M1) for valid approach
$= 2 \times 0.105649839 \times (1 - 0.105649839)$ (A1) for substitution
$= 0.188975901$
$= 0.189$ A1 N3

[3]

(d) (i) 0.327 A2 N2

(ii) 0.0803 A2 N2

(iii) –$1.29 A2 N2

[6]

3. (a) (i) $(6.67, 50.8)$ A2 N2

(ii) $2 < x < 6.67$ A2 N2

[4]

(b) (i) $f'(x) = -3x^2 + 13(2x) - 40(1) + 0$ (A1) for correct derivatives

$f'(x) = -3x^2 + 26x - 40$ A1 N2

(ii) 15 A1 N1

(iii) The equation of the tangent:

$y - f(5) = 15(x - 5)$ M1A1

$y - 36 = 15x - 75$ A1

$15x - y - 39 = 0$ AG N0

[6]

(c) (i) 9 A1 N1

(ii) $\int_2^9 f(x)\mathrm{d}x$ A1 N1

(iii) $\int_2^9 f(x)\mathrm{d}x = \frac{2401}{12}$ A2 N2

[4]

(d) The estimate of $\int_2^9 f(x)\mathrm{d}x$

$= \frac{1}{2}(1.75)\left[\begin{array}{l} f(2) + f(9) \\ +2(f(3.75) + f(5.5) + f(7.25)) \end{array}\right]$ (A2) for substitution

$= \frac{1}{2}(1.75)\left[0 + 0 + 2\left(\begin{array}{l} 16.078125 \\ +42.875 + 48.234375 \end{array}\right)\right]$ (A1) for correct approach

$= 187.578125$

$= 188$ A1 N4

[4]

(e) Underestimate A1 N1

[1]

4. (a) $\frac{\sin A\hat{C}B}{AB} = \frac{\sin A\hat{B}C}{AC}$ (M1) for sine rule

$\frac{\sin A\hat{C}B}{13.9} = \frac{\sin 60.8^\circ}{17.7}$ (A1) for substitution

$A\hat{C}B = 43.27612856^\circ$

$A\hat{C}B = 43.3^\circ$ A1 N3

[3]

(b) The area of the triangle ABC

$= \frac{1}{2}(AB)(AC)\sin B\hat{A}C$ (M1) for area formula

$= \frac{1}{2}(13.9)(17.7)\sin(180^\circ - 60.8^\circ - 43.27612856^\circ)$ (A1) for substitution

$= 119.3212815 \text{ cm}^2$

$= 119 \text{ cm}^2$ A1 N3

[3]

(c) $AB^2 = OA^2 + OB^2 - 2(OA)(OB)\cos A\hat{O}B$ (M1) for cosine rule

$13.9^2 = r^2 + r^2 - 2(r)(r)\cos(2(43.27612856^\circ))$ (A1) for substitution

$13.9^2 = 1.879723687r^2$ (A1) for correct approach

$r^2 = 102.7863836$

$r = 10.13836198$

$r = 10.1$ A1 N4

[4]

(d) The area of sector OAB

$= \pi(10.13836198)^2 \times \frac{2(43.27612856^\circ)}{360^\circ}$ (A1) for substitution

$= 77.63567911 \text{ cm}^2$

$= 77.6 \text{ cm}^2$ A1 N2

[2]

5. (a) 5.5 A1 N1

[1]

(b) $r_s = 0.8982196964$ (A1) for correct value

$r_s = 0.898$ A1 N2

[2]

(c) There is a strong agreement between the two experts. A1 N1

[1]

(d) (i) $a = 0.5610859729$

$a = 0.561$ A1 N1

$b = 11.53846154$

$b = 11.5$ A1 N1

(ii) The estimated percentage

$= 0.5610859729(50) + 11.53846154$ (A1) for substitution

$= 39.59276019\%$

$= 39.6\%$ A1 N2

[4]

(e) (i) $H_1: \mu_x > \mu_y$ A1 N1

(ii) $p\text{-value} = 0.1727476756$ (A1) for correct value

$p\text{-value} = 0.173$ A1 N2

(iii) The null hypothesis is not rejected. A1

As $p\text{-value} > 0.1$. R1 N2

[5]